

# Automated Repair of High Inaccuracies in Numerical Programs

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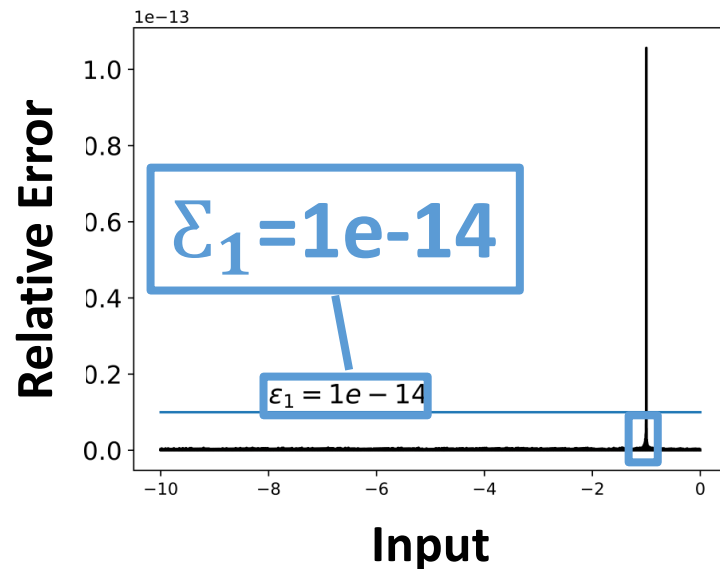
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# Introduction

- High-inaccuracy bug
  - An input  $x$
  - Real arithmetic output  $O_r(x)$  (i.e., mathematical output)
  - Floating-point arithmetic output  $O_f(x)$
  - Threshold  $\varepsilon$

$$\left| \frac{O_r(x) - O_f(x)}{O_r(x)} \right| > \varepsilon$$



# Introduction

## Rounding error

$\mathbb{R}$

0.1

$\mathbb{F}$

0.1000000000  
0055511151...



# Introduction

- Hard to debug and fix high-inaccuracy bugs manually
  - Huge search space (input domain)
    - More than  $9.0e+15$  floating-point (64 bits ) numbers in  $[1,2]$
  - Hard to localize the buggy code
    - Propagation and accumulation of round errors
  - Need of special knowledge on floating-point arithmetic to modify the buggy code

# Introduction

**Automated repair of numerical program:**

**Detecting + Localizing + Repairing  
High-inaccuracy bugs**

# Our Approach

Four phases for automated repair

**Detecting High-inaccuracy Bugs**



**Localizing Buggy Code**



**Generating and Validating Patches**



**Patch Application and Simplification**

# Example

```
double F(double x){  
    //assert(-10<x<100);  
    double y,d,z;  
    z = 0.0;  
    if (x > 0.0){  
        x = pow(x,5);  
        y = x-1.0;  
    }  
    else{  
        d = x*x;  
        y = d-1.0;  
    }  
    while(z < 1e10){  
        z = x*x-y*y;  
        x = x*2.0+1.0;  
    }  
    y = y*z;  
    return y;  
}
```

## Input intervals

- $I_1: [-10.0, 0.0)$
- $I_2: [0.0, 100.0]$

# Our Approach

## Phase I: Detecting High-inaccuracy Bugs

- Obtaining (approximate) mathematical output
  - Shadow value execution in higher precision (64bits to 128 bits) (FPDebug) [Benz '12]
- Detecting algorithm to find negative test cases
  - Locality-Sensitive Genetic Algorithm (LSGA) [Zou '15]
  - Binary Guided Random Testing (BGRT) [Chiang '14]



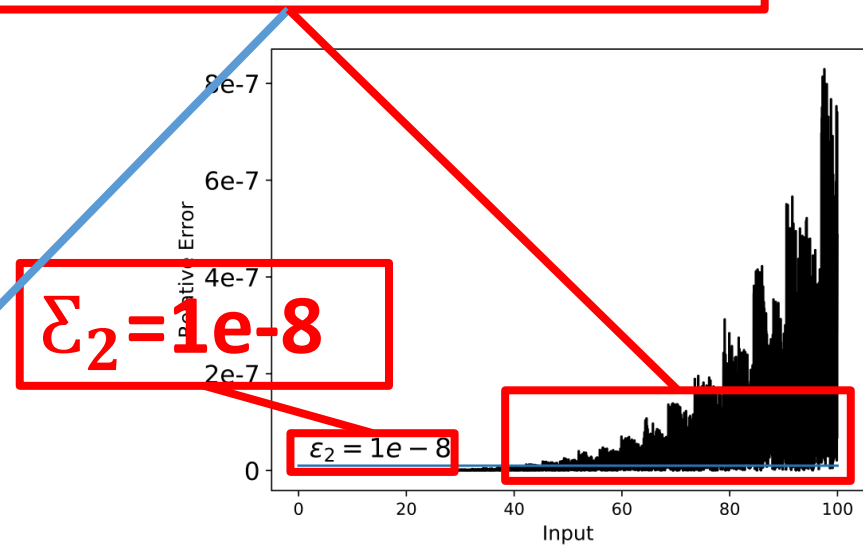
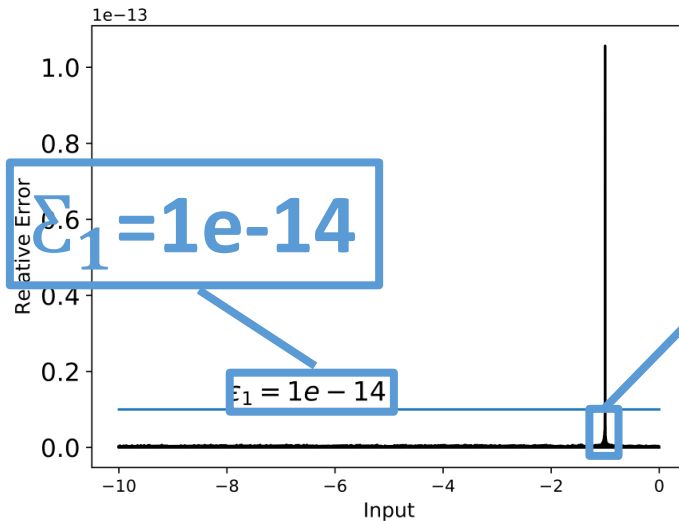
# Our Approach

## Phase I: Detecting High-inaccuracy Bugs

- Using FPDebug to approximate the real arithmetic results and Binary Guided Random Testing to search inputs.

### Input intervals triggering bugs

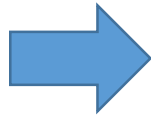
- $I_1: x \in [-1.0042, -0.9982]$
- $I_2: x \in [39.5303, 100.0000]$



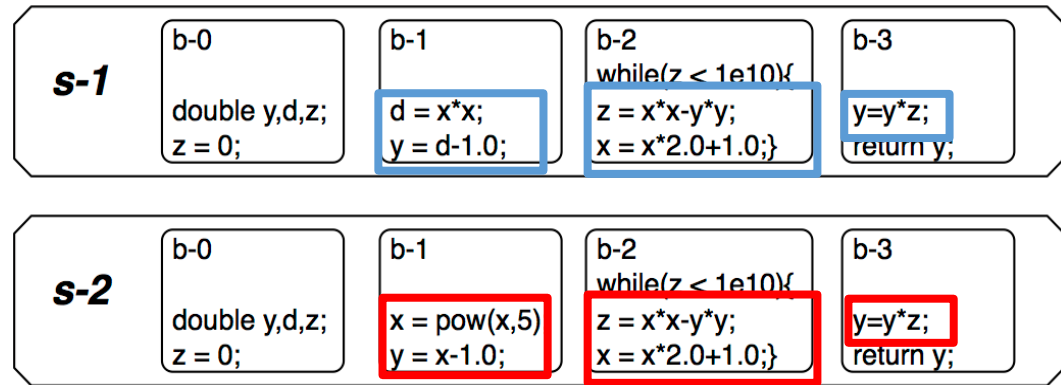
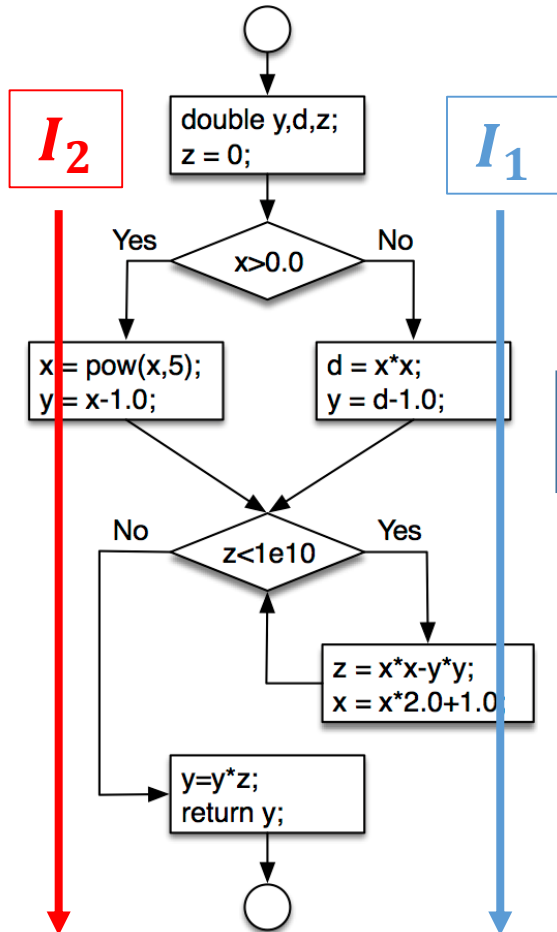
# Our Approach

## Phase 2: Localizing buggy code

control flow graph



Slices and Blocks



### Input intervals triggering bugs

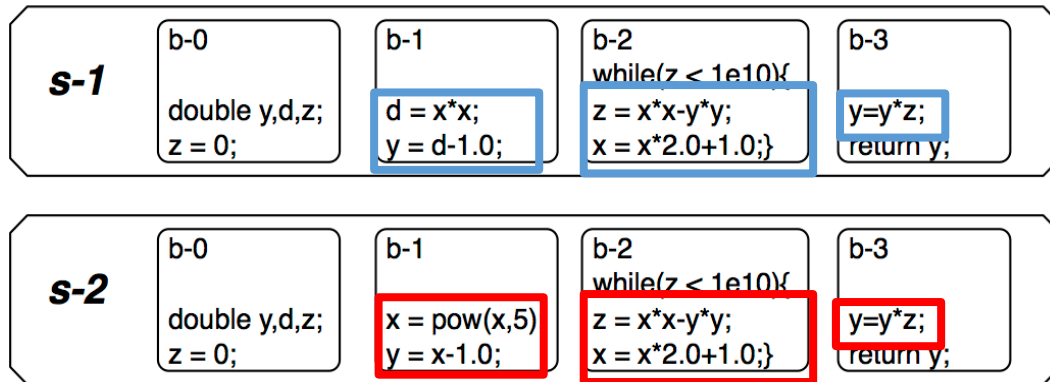
- $I_1: x \in [-1.0042, -0.9982]$
- $I_2: x \in [39.5303, 100.0000]$

# Our Approach

## Phase 2: Localizing buggy code

- Ranking blocks according to the relative error that each block introduces

### Slices and Blocks



### Ranking list of Blocks

<i>s-1</i>	<i>s-2</i>
<i>b-1</i>	<i>b-2</i>
<i>b-2</i>	<i>b-3</i>
<i>b-3</i>	<i>b-1</i>

# Our Approach

## Phase 3: Generating and Validating Patches

- **Generating patches**
  - **symbolical calculation**
  - **mathematically equivalent transformation**

*s-l: b-l*

$$\begin{array}{l} d = x * x \\ y = d - 1.0 \end{array}$$

$$\begin{array}{l} d = x * x \\ y = x * x - 1 \end{array}$$

$$\begin{array}{l} d = x * x \\ y = (x - 1) * (x + 1) \end{array}$$

symbolical  
calculation

mathematically  
equivalent transformation

# Our Approach

## Phase 3: Generating and Validating Patches

- Validating Patches
- Regression testing

```
d = x*x;  
y = d-1.0;
```



```
if ((x >= -1.0042)  
    &&(x < -0.9982)){  
    d = x*x;  
    y = (x-1.0)*(x+1.0);  
}else{  
    d = x*x;  
    y = d-1.0;}
```

Input intervals trigger bugs

- $I_1: x \in [-1.0042, -0.9982]$
- $I_2: x \in [39.5303, 100.0000]$

```
while(z < 1e10){  
    z = x*x-y*y;  
    x = x*2.0+1.0;  
}
```



```
if ((x >= 35.5303)  
    &&(x <= 100)){  
    while(z < 1e10){  
        z = (x-y)*(x+y);  
        x = x*2.0+1.0;  
    }else{  
        while(z < 1e10){  
            z = x*x-y*y;  
            x = x*2.0+1.0;  
        }  
    }  
}
```

# Our Approach

## Phase 4: Patch Application

```
double F(double x){
  //assert(-10<x<100);
  double y,d,z;
  z = 0.0;
  if (x > 0.0){
    x = pow(x,5);
    y = x-1.0;
  }
  else{
    d = x*x;
    y = d-1.0;
  }
  while(z < 1e10){
    z = x*x-y*y;
    x = x*2.0+1.0;
  }
  y = y*z;
  return y;
}
```



```
double F(double x){
  //assert(-10<x<100);
  double y,d,z;
  z = 0.0;
  if (x > 0.0){
    x = pow(x,5);
    y = x-1.0;
  }
  else{
    if ((x>= -1.0042)
      &&(x<-0.9982)){
      d = x*x;
      y = (x-1.0)*(x+1.0);
    }else{
      d = x*x;
      y = d-1.0;}
  }
  if ((x>=35.5303)
    &&(x<=100)){
    while(z<1e10){
      z = (x-y)*(x+y);
      x = x*2.0+1.0;
    }else{
      while(z < 1e10){
        z = x*x-y*y;
        x = x*2.0+1.0;
      }
    }
  }
  y = y*z;
  return y;
}
```

# Our Approach

## Phase 4: Patch Simplification

```
if ((x >= -1.0042)
    &&(x < -0.9982)){
    d = x*x;
    y = (x-1.0)*(x+1.0);
}else{
    d = x*x;
    y = d-1.0;}
```



```
d = x*x;
y = d-1.0;
```

```
if ((x >= 35.5303)
    &&(x <= 100)){
    while(z < 1e10){
        z = (x-y)*(x+y);
        x = x*2.0+1.0;
    }else{
        while(z < 1e10){
            z = x*x-y*y;
            x = x*2.0+1.0;
        }
    }
```

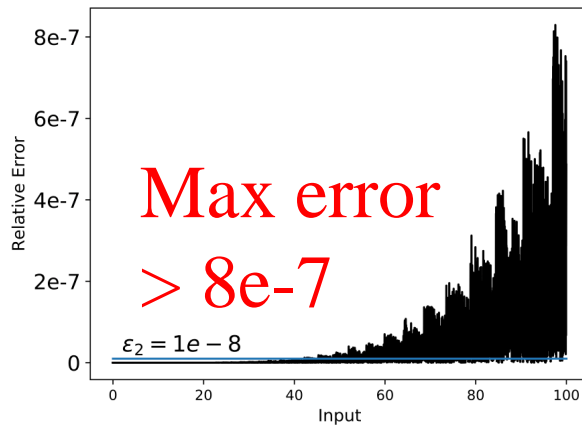
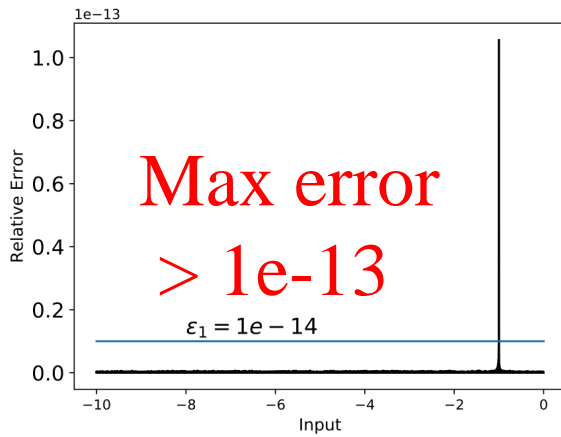


```
while(z < 1e10){
    z = (x-y)*(x+y);
    x = x*2.0+1.0;
}
```

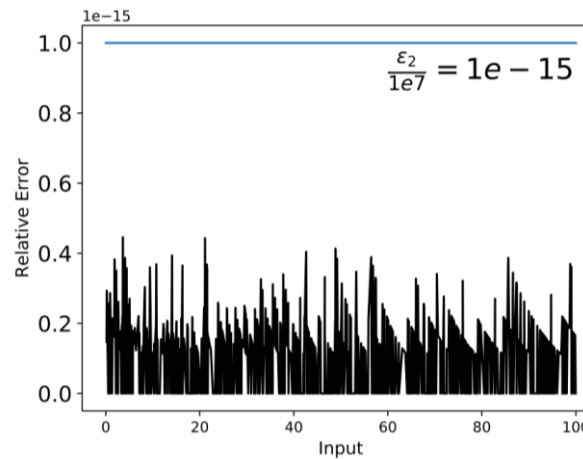
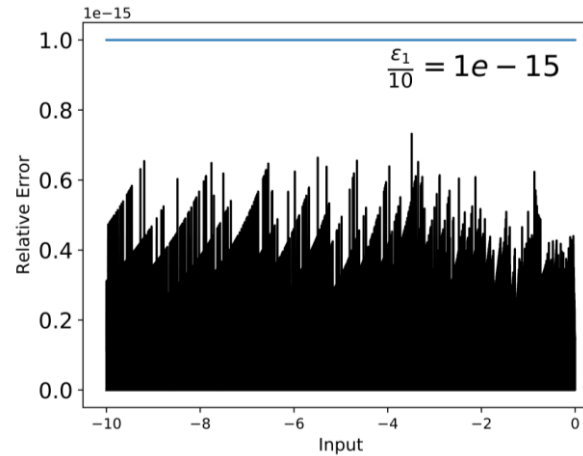
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double F(double x){
    //assert(-10 < x < 100);
    double y,d,z;
    z = 0.0;
    if (x > 0.0){
        x = pow(x,5);
        y = x-1.0;
    }
    else{
        d = x*x;
        y = (x-1)*(x+1);
    }
    while(z < 1e10){
        z = (x-y)*(x+y);
        x = x*2.0+1.0;
    }
    y = y*z;
    return y;
}
```

# Our Approach

## Before repair



## After repair



Max error  
< 1e-15



# Experiments

Program	Input Domain	Time(s)			Max. Relative Error	
		Time for Detecting	Time for Patches	Total Time	Before Repair	After Repair
frac2	(0,1e5]	120.22	5.06	125.29	1.38E-11	9.33E-17
frac3	(1,200]	75.54	14.87	90.41	4.80E-12	1.46E-16
sqrt2	(0,1e7]	123.71	5.04	128.76	1.43E-09	1.53E-16
sqr2	(0,1e10]	217.94	3.11	221.05	7.87E-07	0.00E+00
rsqrt	(0,700]	93.76	9.58	103.35	2.33E-13	2.64E-16

**Benchmark: 5 programs from FPBench (a benchmark for floating point analysis [Damouche '16])**

# Conclusion

- Propose a novel approach for automatically detecting, localizing, and repairing high-inaccuracy bugs in numerical programs
- Develop an automated repair prototype tool , evaluate it on several benchmark programs and achieve promising results

# Future Work

- Design more efficient detecting algorithm to find negative test cases
- Improve our tool and evaluate it on real-world scientific numerical programs, e.g., the GNU Scientific Library (GSL)

**Thank you!**